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Variational Monte Carlo calculation of binding energy for ${}^{4}_{\Sigma}$He hypernucleus

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Abstract : The binding energy of ${}^{4}_{\Sigma}$He hypernucleus has been calculated variationally with the (Σ+p+p+n) four-body model. Calculations were done using Monte Carlo technique adopting a Gaussian type potential The calculated value of the binding energy 3 73 MeV compares well with the experimental value $3.2^{+0.4}_{-1.4}$ MeV.

Keywords : Monte Carlo calculation, binding energy calculation, ${}^{4}_{\Sigma}$He hypernucleus

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1. Introduction

The first evidence of narrow Σ-hypernuclear states was found from the CERN group using the (K^-, π^+) reaction in flight in 1982. The peaks, which had about 8 MeV width, lie above the Σ-emission threshold. In spite of many theoretical efforts, it still remains a puzzle as to why such narrow peaks exist in the Σ-continuum region. Narrow peaks were also observed elsewhere. In the kaon in-flight experiments of ^{6}Li, ^{9}Be, ^{12}C and ^{16}O, peak structures with widths of about 5 MeV have been observed at excitation energies ε_{Σ} = 2–10 MeV. In a stopped kaon experiment [1] on ^{12}C with momentum of about 170 MeV/c three peaks have been reported with widths as small as 3 MeV. Gal *et al* [2] proposed a possible explanation of the narrowness of the Σ-formation peaks in terms of an unstable bound state, but this is not generally accepted [3,4]. The problem, thus, is not solved. After the discovery of the first Σ-hypernuclear structures in (K^-, π) reactions by the CERN group [5], a lot of interest

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developed on the subject. The Σ-nucleus interactions are not well understood even today despite a number of experimental and theoretical efforts [6].

An important source of obtaining information about the interaction is the binding energy (BE) calculations. Such calculations were done for the λ-hypernuclei with the idea of obtaining information about the (λ–λ) interaction. Among the different approaches for the BE calculations, the variational calculation procedure has obtained quite a good place. Dalitz *et al* [7] and Bodmer *et al* [8] carried out variational calculations of some *s*-shell model hypernuclei and a few *p*-shell hypernuclei [9] using appropriate models. Lomnitz-Adler *et al* [10] used Reid potential and ignored interaction in ${}^{4}_{\lambda}$He during variational calculation.

The Monte Carlo (MC) methods were used to evaluate integrals in references [11–13]. The double hypernucleus ${}^{31}_{\lambda\lambda}$Si has been considered as a 3-body system (λ-λ-^{29}Si) in the work of Ahsan and Ali [14], who later did some 4-body calculations [15] as well for the same nucleus using the MC technique. Recently a 4-body variational calculation has been performed by Alam *et al* [16,17] for the BE difference of the hypernuclear isodoublet ${}^{4}_{\lambda}$He $-$ ${}^{4}_{\lambda}$H. The prediction showed excellent agreement with the experimental results. This work inspired us for considering the present problem to see the extent to which the calculation procedure may be adopted for a Σ-hypernucleus. For the purpose, ${}^{4}_{\Sigma}$He hypernucleus has been chosen. The nucleus is an interesting target because it allows us to gather information on the $T = 1/2$ and $T = 3/2$ channels. Also it is a relatively simple nucleus and thus few body calculations may be performed to study the structure of ${}^{4}_{\Sigma}$He and obtain relevant information.

2. The binding energy equation

In variational technique of BE calculation one should always try to evaluate the minimum value of E, where

$$E = \left(\int \Psi^{*} H \Psi d\tau\right) \Big/ \left(\int \Psi^{*} \Psi d\tau\right) \tag{1}$$

Here Ψ denotes the trial wave function and H, the Hamiltonian of the 4-body system. The wave function of the concerned hypernucleus may be formed as the product of an orbital wave function and a spin wave function. Considering that the particle stays outside the core its orbital wave function can be used to describe the motion of the particle with respect to the core of the nucleus. For simplicity of the problem, and also for practical reasons it is assumed that the core nucleus suffers relatively little distortion [18].

In the work, the wave function has been expressed in terms of the interparticle distances, r_{ij} where $i < j$ and $i = 1, 2, 3, 4$ and $j = 2, 3, 4, 5$. The interparticle distances must be less than the hardcore radius, which implies that the wave function will vanish at distances larger than the radius of the nucleus. This situation directs us to consider

12 position coordinates for the 4 particles (3 are of the centre of mass, 3 are of Euler angles involving the orientation of the hypernuclear space, and the other 6 describe the tetrahedron

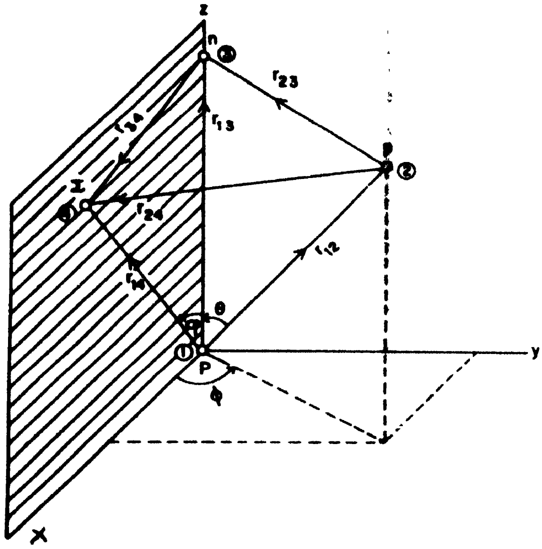

Figure 1. The interparticle distances for the hypernucleus ${}^4_\Sigma$He.

formed by the 4 particles). For this problem, we have considered 3 interparticle distances satisfying the triangular inequality as shown in Figure 1. The Laplacian operator corresponding to the i-th nucleon can now be written in terms of the displacement vector r_{ij} as :

$$\sum \nabla_i^2 \Psi_s = 2\sum\left(\partial^2\Psi_s/\partial^2 r_{ij} + 2\partial\Psi_s/r_{ij}\partial r_{ij}\right)$$
$$-2\sum\left(r_{ij}\ r_{ik}/r_{ij}r_{ik}\right)\left(\partial^2\Psi_s/\partial r_{ij}\partial r_{ik}\right) \quad (2)$$

Following the 4-body approaches of Ahsan and Ali [14] the shape of the correlation function is taken as $f(r) = \exp\left(-a_k r_{ij}^2\right) + b_k \exp\left(-c_k r_{ij}^2\right)$. The first term represents the long-range part of the wave function; the second term represents the wave function of the 2-body systems in the region of close approach.

Finally, the wave function can be written as the product of the 6 correlation functions as

$$\Psi = f(r_{12})f(r_{13})f(r_{14})f(r_{23})f(r_{24})f(r_{34})$$

A product type wave function related to a correlation function of the above form was found useful in earlier works on 3-body [14] and on 4-body [15] hypernuclear systems. If 1, 2, 3 and 4 are the positions of the nucleons p, p, n and Σ respectively then their corresponding interparticle distances (Figure 1) between pp, pn, $p\Sigma$, pn, $p\Sigma$, $n\Sigma$ are r_{12}, r_{13}, r_{14}, r_{23}, r_{24} and r_{34} respectively. The Hamiltonian of the system is given by

$$H = -\sum_{i=0}^{3}\left(\hbar^2/2\mu_i\right)\nabla_i^2 + \sum_{i=0}^{2}\sum_{j=i+1}^{3} V_{ij}$$

In the above equation μ_i represents the reduced mass of the 4 pairs of particles and have the appropriate numerical values. ∇_i^2 is the Laplacian operator corresponding to the i-th particle and r_{ij} is the vector displacement from i-th to the j-th particle. Using the correlation function as described above the expression for $\nabla^2\Psi$ may be found out. This has been calculated elsewhere [15,16] and is given by

$$\begin{aligned}
\nabla^2\Psi = {} & 2\sum_{i=0}^{2}\sum_{j=i+1}^{3}\Big[2a_{ij}\left(2a_{ij}r_{ij}^2-1\right)\exp\left(-a_{ij}r_{ij}^2\right) \\
& + 2b_{ij}c_{ij}\left(2c_{ij}r_{ij}^2-1\right)\exp\left(-c_{ij}r_{ij}^2\right)\Big]\prod_{k\neq j} f_{ik} \\
& + \sum_{i=0}^{2}\sum_{j=i+1}^{3} - 4\left[a_{ij}\exp\left(-a_{ij}r_{ij}^2\right)\right] + b_{ij}c_{ij}\exp\left(-c_{ij}r_{ij}^2\right)\prod_{k\neq j} f_{ik} \\
& + 2\sum_{i=0}^{2}\sum_{j=i+1}^{3}\sum_{i_1=i}^{2}\sum_{j_1=i_1+1}^{3}\left(r_{ij}\cdot r_{ij}\right)\Big\{a_{ij}\exp\left(-a_{ij}r_{ij}^2\right) \\
& + b_{ij}c_{ij}\exp\left(-c_{ij}r_{ij}^2\right)\Big\} \times \Big\{a_{i_1j_1}\exp\left(-a_{i_1j_1}r_{i_1j_1}^2\right) \\
& + b_{i_1j_1}c_{i_1j_1}\exp\left(-c_{i_1j_1}r_{i_1j_1}^2\right)\Big\}\prod_{\substack{k\neq i_1\\ k\neq j_1}} f_{ik}, \quad i<j
\end{aligned} \tag{3}$$

As was done in reference [15] the volume integral of equation (1) has been considered as

$$d\tau = 16\pi^2\prod_{\substack{i=0\\ j=i+1}}^{\substack{i=2\\ j=3}} r_{ij}dr_{ij} \Big/ \left[r_{02}\left(r_{13}^2 - r_{13(\min)}^2\right)^{1/2}\left(r_{13(\max)}^2 - r_{13}^2\right)^{1/2}\right] \tag{4}$$

where $\quad r_{13(\min)}^2 = r_{03}^2 + r_{01}^2 - 2r_{03}r_{01}\cos(\theta-\Psi)$

$\quad r_{13(\max)}^2 = r_{03}^2 + r_{01}^2 - 2r_{03}r_{01}\cos(\theta+\Psi)$

3. The Σn and nn interactions

3.1. The Σn potential :

The Σn potential considered in this work was of the form :

$$\hat{V}_{\Sigma n} = V_{(3/2)1}\hat{P}_{(3/2)1} + V_{(3/2)0}\hat{P}_{(3/2)0} + V_{(1/2)1}\hat{P}_{(1/2)1} + V_{(1/2)0}\hat{P}_{(1/2)0}$$

where $\hat{P}_{TS}$ is the projection operator to the Σn isospin (T)-spin (S) state. The radial form V_{TS} of the potential is given by a 2-range Gaussian form

$$V_{TS} = v_C \exp\left\{-\left(r/a_C\right)^2\right\} + (v_A + iW_A)\exp\left\{-\left(r/a_A\right)^2\right\}$$

Here the imaginary part describes the $\Sigma n \to \Lambda n$ conversion process. Other values taken were as follows :

$$v_C = 5000 \text{ MeV}, \quad v_A = -179.2 \text{ MeV},$$

$$a_C = 0.4\, fm \quad \text{and} \quad a_A = 1.1\, fm$$

The complex Σn potential was taken from the work of Harada *et al* [19] who constructed the complex potential in order to eliminate the Λn channel from the microscopic 4-body calculation of ${}^4_\Sigma$He hypernuclear bound state.

3.2. The nn potential :

The *nn* interaction employed is of central Gaussian type :

$$\hat{V}_{nn} = v_1 \exp\left\{-\left(r/a_1\right)^2\right\} - v_2 \exp\left\{-\left(r/a_2\right)^2\right\} - v_3 \exp\left\{-\left(r/a_3\right)^2\right\}$$

where $v_1 = 2000$ MeV, $v_2 = 270$ (366) MeV for the ${}^1E({}^3E)$ state, $v_3 = 5$ MeV, $a_1 = 0.447$ fm, $a_2 = 0.942$ fm and $a_3 = 2.5$ fm. These values were taken from the work of ${}^4_\Sigma$He hypernuclear bound state done by Harada *et al* [19].

4. The Monte Carlo calculation

MC method has been applied to evaluate the integrals of the BE. Eq. (1). The wave function (WF) had to be made zero within and at the surface of the core. This boundary condition together with the addition of antisymmetry condition made an analytical evaluation of the expression (Eq. 1) very complicated. During the calculation of the integrations for BE of ${}^4_\Sigma$He nucleus the computing system had to follow antisymmetrization and then performed a random process of computing technique.

The random numbers U in the domain (0, 1) were generated so as to satisfy the equation $U = \exp\left(-a_{01} r_{01}^2\right)$. All radius vectors $\boldsymbol{r}_{ij}$ were generated in this way. The weight functions w_1, w_2 for the numerator and denominator of eq. (1) respectively defined as $w_1 = (1/p)(\Psi^* H \Psi)$ and $w_2 = (1/p)(\Psi^* \Psi)$ were then calculated for a large number of times n. The average values were then found out to estimate the integrals of eq. (1).

During the calculations initially the WF parameter values were guessed from information collected from literature. A_1 was varied keeping all other parameter values unchanged. The variation was plotted. Figure 2, being a representative one, shows the result of such a variation. The value of a_1 which gave the minimum value of BE was identified.

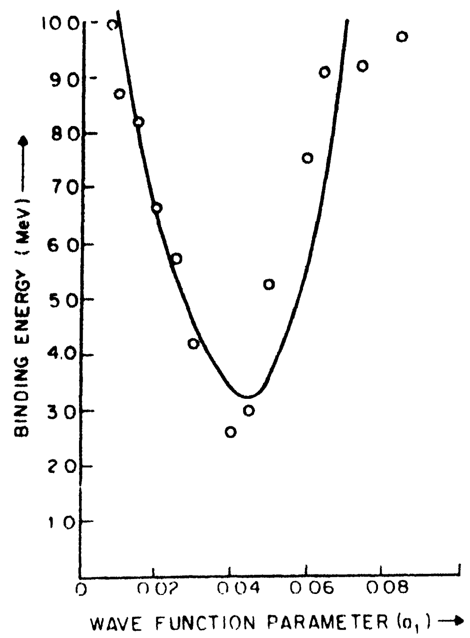

Figure 2. Variation of binding energy with wave function parameter a_1.

This value was then kept constant for the subsequent calculations. Similarly values of a_2–a_6, b_1–b_6, c_1–c_6 were found out.

5. Results and discussion

5.1. Checking the validity of the MC programme :

Before application, the programme was first applied to the case of ${}^{31}_{\lambda\lambda}$Si to check its validity taking the same potentials as were taken by Ahsan *et al*, and making similar assumptions *e.g.*, 4-body case *etc.* In both the cases variational procedure was adopted. Finally we got the same result as that of Ahsan *et al* [15]. Alam *et al* [17] also used this programme during their calculations of BE on ${}^{4}_{\lambda}$He and ${}^{4}_{\lambda}$H; and found a good agreement between their results with other theoretical results obtained by different techniques. These confirmed that our MC programme was valid for the purpose of BE calculation.

5.2. Binding energy variation with variation of wave function parameters :

In the present work, variation of BE with the variation of the wave function parameters was investigated elaborately to find the sensitive ones among a_2–a_6, b_1–b_6 and c_1–c_6 parameters These were then included in the calculation. The WF parameters a_1, a_2, a_4 and a_6 showed some effects on the variation of BE, the graph being approximately U-shaped (Figure 2). There was a random variation of BE with the variation of the WF parameters a_3 and a_5. The other parameters such as b_1–b_6 and c_1–c_6 showed no such variation. BE values in such cases were found constant (approximately 5.23 MeV). For these, of course, the parameters a_1–a_6 were kept constant at their optimum values.

5.3. *Optimum parameter values :*

The optimum parameter values for the potential parameters used in the present work are shown in Table 1. These parameters may be used as a guideline for similar work in future.

Table 1 The optimum wave function parameter values.

Parameters	Values of the parameters	Parameters	Values of the parameters
a_1	0.040	b_4	6 000
a_2	0.100	b_5	6 000
a_3	0.035	b_6	6 000
a_4	0.300	c_1	0.200
a_5	0 005	c_2	0.120
a_6	0 100	c_3	0 120
b_1	5 000	c_4	0 120
b_2	5.000	c_5	0 120
b_3	6 000	c_6	0 120

5.4. *Comparison of the present and the past results :*

The calculated result of BE for ${}^4_\Sigma$He hypernucleus is found 3.73 MeV in the present work. This result compares well with that of Harada *et al* [19] who found the BE value 4.6 and 3.7 MeV for 2 different potentials SAP-1 and SAP-2 respectively using coupled channel approach. Both of these results again compare well with the experimental result of 3.2 MeV obtained through ^{4}He (stopped K^-, π^-) reaction at KEK [20].

The nucleus could be considered as a 3-body system in which case the BE calculation would be simplier, and the result would possibly be less accurate. In support of these assumptions, the 2 analyses of Ahsan *et al* for the BE calculation for ${}^{31}_{\lambda\lambda}$Si may be quoted. The system ${}^{31}_{\lambda\lambda}$Si was considered as a 3-body system $\lambda + \lambda + {}^{29}\mathrm{Si}$(14) first and subsequently a 4-body system ${}^{28}\mathrm{Si} + n + \lambda + \lambda$(15). The assumptions of first case, being less accurate than the later one, gave also less accurate result : 41.54 MeV as against 39.90 MeV for the 4-body assumption. The experimental value of the BE is 38.2 ± 6.3 MeV.

The Σ-nucleus and Σn potentials are both taken to be of Gaussian form in the present work. This is again a good assumption, and for some practical reasons this form is widely used [21]. This potential depth is also chosen against its already proven success in explaining some physical phenomenon in the nuclear realm.

5.5. *Σn potential :*

In our present work, we have done microscopic 4-body $ppn\Sigma$ calculations using MC method with realistic $n\Sigma$ and nn interactions of Gaussian type potentials. Harada *et al* [19] considered Σn potential of the complex form (in case of SAP-1, which has also been considered in the present work,) as given in eq. (2) during their 4-body Σnnn calculations using coupled channel approximation. The Σn potential shows some characteristic features. It has a strong Lane term and a repulsive bump at short distances. This Lane term plays an important role to make the nucleus-Σ system bound and recovers the isospin symmetry. In the work of Harada *et al* [19] the $T = 1/2$ state was found to dominate almost completely in the nucleus-Σ interaction region, while the $T = 3/2$ component is mixed at large distances. The calculated probability for total isospin $T = 1/2$ found were 0.995 and 0.992 respectively for SAP-1 and SAP-2 potentials which results in total isospin $T = 1/2$ as a good quantum number for ${}^{4}_{\Sigma}$He. Thus the ${}^{4}_{\Sigma}$He-hypernucleus stays in the $T = 1/2$ state and dresses the Σ^{+} particle at the nuclear surface. In the $T = 3/2$ state the potential becomes strongly repulsive in the real part and very weak in the imaginary part.

5.6. Σ-nucleus potentials :

Housemann *et al* [22] reported that the gross feature of the Σ formation spectra and the energy integrated absolute cross section can be reproduced by assuming a very weak attractive Σ-nucleus central portential with a strength $V_{\Sigma}^{(0)}$ between –5 and –10 MeV. The Σ-nucleus central potential depth (= –6 MeV) considered in the present work falls clearly within the limits.

6. Conclusion

In the present work ground state BE has been calculated for the ${}^{4}_{\Sigma}$He hypernucleus considering Gaussian type potential. An investigation of BE variation with the variation of all the WF parameters a_1–a_6, b_1–b_6 and c_1–c_6 taking one at a time has been done in detail to find out the sensitive parameters. Of these 18 parameters, the parameters a_1, a_2, a_4 and a_6 were found to have some effects on the variation of BE, the graph showing the variation being approximately U-shaped. a_3 and a_5 showed a random variation and other parameters such as b_1–b_6 and c_1–c_6 showed no effect on the variation of BE. A set of optimum values has been obtained in the work, which corresponds to the ground state BE. The parameter set is given for reference for future study.

The SAP-1 potential set of Harada *et al* [19] has been considered for the present study keeping the approach different. As against a coupled channel calculation done by Harada *et al*, the present calculations have been done following MC variational approach. The calculated BE value obtained in the present work is 3.73 MeV which compares well with that of Harada *et al*. The authors found the BE values 4.6 and 3.7 MeV for 2 different potentials SAP-1 and SAP-2 respectively. Both of these results again compare well with the experimental BE result of 3.2 MeV obtained through ^{4}He (stopped K^{-}, π^{-}) reaction at KEK [20].

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